# Day 3 - Statistical Inference
# Part I

1. Introduction to hypothesis testing and confidence intervals
2. One-sample Student's test and confidence interval
3. Paired data, two independent samples

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# Statistical Inference, part I: Overview

## Schedule for today: Lectures in flipped classroom style

- 12:45 – 13:30 Introductory lecture
- 13:30 – 14:15 Self study session*
- 14:15 – 15:00 Group work session*
- 15:00 – 15:30 Closing session (Wrap-up / Q&A)

[*you should take a break at some point in these two stretches]

## Tomorrow morning: Lab in flipped classroom style

- One-sample t-Student test and confidence interval with **R**
- Two-sample t-Student test and confidence interval with **R**

# Statistical Inference, part I: Introductory Lecture

- Shortly explain the **key concepts** for today
- Make clear **what to focus on**
- Explain how to use the **learning material** in the **Self study session**
- Give **guiding questions** for the **Group work session**

# Statistical Inference, part I: Introductory Lecture

## Key concepts

1. **Properties of the Sample Mean**
   - Recap from Day 1
   - Standard deviation vs. standard error
2. **Confidence Intervals (CIs)**
   - The Student t-distribution
   - Confidence interval for the mean
3. **Testing an hypothesis**
   - One-sample t-test
   - Paired data
   - Two sample t-test

# Key concept 1. Properties of the Sample Mean

## Recap from Day 1

How to describe the data distribution?

- **central measures** (mean, median, mode)
- **measures of variation** (range / IQR, empirical variance / standard deviation)

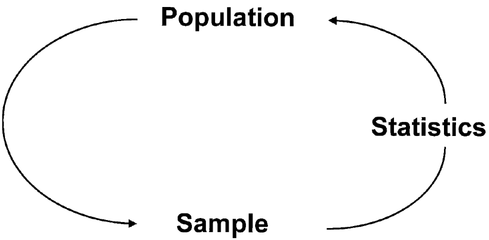
## Some facts:

- which measure to use depends on the situation
- the sample mean $\bar{X}$ is an estimator for the **population** mean $\mu$
- the empirical standard deviation $s$ is an estimator for the population standard deviation $\sigma$
- the sample mean $\bar{X}$ is a **normally distributed random variable**, with mean $\mu$ and standard deviation $\sigma/\sqrt{n}$, the latter is also called **standard error**

# Key concept 1. Properties of the Sample Mean

## Recap from Day 1

- **Inferential Statistics** is about using information from a sample (data set) to **make inference** about the population it originates from
- Therefore, the **sample** is of interest for what it tells the investigator about the **population** which it represents



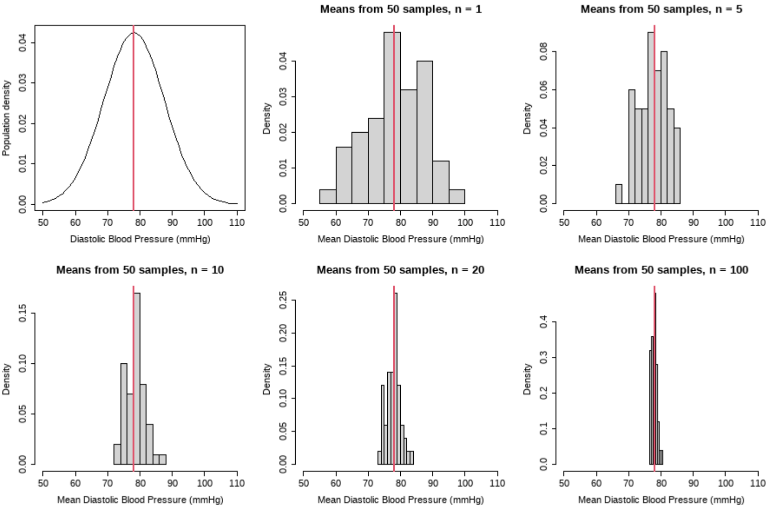
# Key concept 1. Properties of the Sample Mean

## standard deviation vs. standard error

- every sample will give a different estimate of $\bar{X}$ due to **sample variation**
- the **standard error** of the sample mean reflects this variation, as it measures *how precisely the population mean* $\mu$ *is estimated by the sample mean* $\bar{X}$
- **by construction,** the standard error decreases when the sample size $n$ increases (also natural / intuitive!)

# Key concept 1. standard deviation vs. standard error

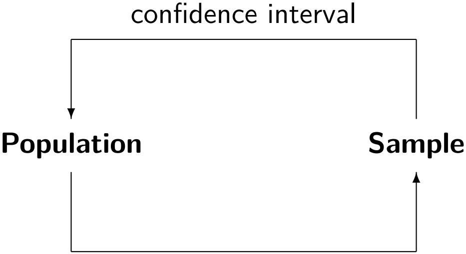
Example 4.4 at page 39-41 of K&S → **simulated in R!**



# Key concept 2. Confidence Intervals (CIs)

## Confidence intervals for the mean

- The sample mean $\bar{X}$ is an **estimate** of the true mean $\mu$ in the whole population
- We seek a method to **quantify how representative our estimate is**
- We are able to construct a **range of likely values**, called a **confidence interval** (CI), for the (unknown) population mean based on the sample mean and its standard error



# Key concept 2. Confidence Intervals (CIs)

## 95%-confidence interval

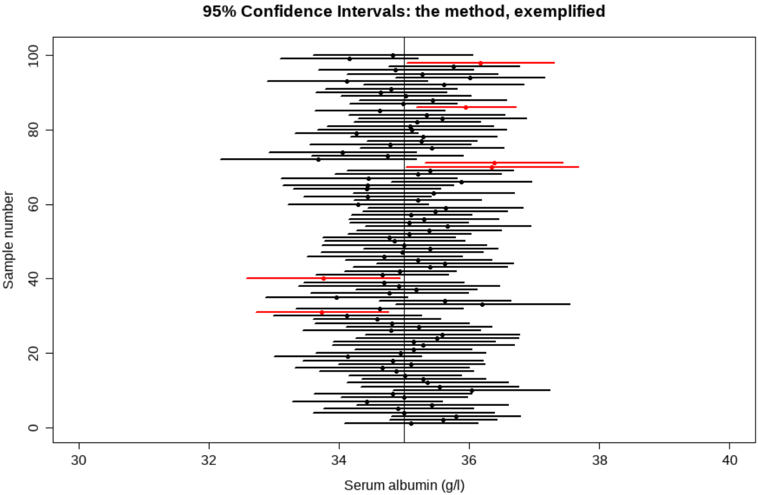
- A method that we can apply to the sample to produce an interval
- The probability that this method will produce an interval that contains the true value is 95%
- We will refer to such interval as 95% *confidence interval*

## common misunderstanding

this is not the same as saying that our estimated interval contains the true value with 95% probability!

# Key concept 2. Confidence Intervals (CIs)

**Simulation in R:** Confidence intervals for the **mean serum albumin** constructed from 100 random samples of size $n = 25$. Vertical line at the population mean $\mu$; red CIs do not cover $\mu$



# Key concept 3. Testing an hypothesis

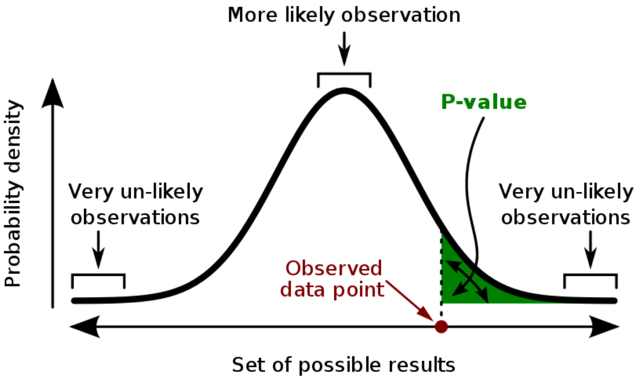
## Hypothesis testing in general

- State your **null hypothesis** $H_0$: **aim** of the test is to check whether the data provide **sufficient evidence to reject it**
- Derive the **test statistic**, who has a certain distribution
- **Take a decision:** accept/reject, or calculate the **p-value** (There is a relation between the two strategies: if the p-value is below a certain level you can reject $H_0$)

## P-value: definition

The probability that the observed result, or a result more extreme, is true, given $H_0$ is true.

# P-value


More likely observation
P-value
Probability density
Very un-likely observations
Very un-likely observations
Observed data point
Set of possible results

# Type I and Type II errors

| HYPOTHESIS TESTING OUTCOMES | | Reality | |
|---|---|---|---|
| | | The Null Hypothesis Is True | The Alternative Hypothesis is True |
| Research | The Null Hypothesis Is True | Accurate $1 - \alpha$ | Type II Error $\beta$ |
| | The Alternative Hypothesis is True | Type I Error $\alpha$ | Accurate $1 - \beta$ |

- **Type I error** (false positive):
  P($H_0$ rejected | $H_0$ true) $= \alpha$
  Also called **level** of the test, as it defines the test itself
  ($\alpha$ is thus determined in advance, example value 5%)
- **Type II error** (false negative):
  P($H_0$ not rejected | $H_0$ false) $= \beta$
  Influenced by sample size; it is equal to 1 - **power**

## Test procedure

1. Formulate the test (null hypothesis & alternative hypothesis)
2. Choose an appropriate test and level $\alpha$
3. Calculate the test-statistic
   - Compare the test-statistic with the $\alpha$-threshold, **OR**
   - Calculate the p-value, and compare it with $\alpha$
4. Decide whether the null hypothesis is to be rejected or not: **reject if**
   - the test-statistic **is larger than** the $\alpha$-threshold, **OR**
   - the p-value **is smaller than** $\alpha$
5. Formulate the conclusion

## Correspondence between CI and test

If the 95%-CI for $\mu$ does not include $\mu_0$, then the corresponding test can be rejected at the 5% level

# Summary

## Key terms and concepts

- Recap from Day 1 (concepts from **Descriptive Statistics**)
- **Inferential Statistics** (as opposed to descriptive statistics)
- **Population** and **sample**, properties of the **sample mean**
- Standard deviation vs **Standard error** of the mean
- **Confidence intervals:** general idea, concept of **coverage**
  - CI for the mean when $\sigma$ is known
  - CI for the mean when $\sigma$ is unknown, the Student t-distribution
  - Non-normality, small sample sizes
  - CI for the mean difference $\mu_1 - \mu_0$ of two independent samples
- **Testing an hypothesis:** general idea, concept of **p-value**
  - one-sample t-test for the population mean
  - test for paired data
  - two-independent-samples t-test for the mean difference $\mu_1 - \mu_0$

## Self study session – Tasks

1. **Deepen your understanding** of each **key concept** from the previous slides by reading the corresponding longer slides:
   - `day3_key_concept_1.pdf`
   - `day3_key_concept_2.pdf`
   - `day3_key_concept_3.pdf`
2. **Verify your learning outcome:**
   - **Review the Summary** (slide 16, “Key terms and concepts”) in this presentation, and make sure you understand all terms
   - **IF** you feel you are still not familiar with any terms and concepts from the summary slides, then
     - use the provided **Learning Material** (next slide) to read more
     - ASK ME!! (I will be in class)
3. **Prepare for the group work session** by keeping in mind the “Guiding questions for the group work session” (slide 19) when reviewing the material

# Self study session

## Learning Material

- Properties of **the mean**: Aalen chapter 8.1, Kirkwood and Sterne (K&S) chapter 4
- **CIs for the mean**: Aalen chapter 8.3, K&S chapter 6 (Student t-distribution: Aalen chapter 8.2)
- One sample **t-test**: Aalen chapter 8.4
- Paired data: Aalen chapter 8.5, K&S chapter 7
- Two sample t-test: Aalen chapter 8.6, K&S chapter 7
- **General discussions** on the use of p-values and confidence intervals for interpreting results: K&S chapter 8

# Group work session

## Task

In your group (which should include 4-6 participants), jointly **revise the following guiding questions and provide an answer**

## Guiding questions

1. What is the property of the sample mean that allows us to build Confidence Intervals and Hypothesis Tests?
2. Which is the relationship between a Confidence Interval and a Hypothesis Test, and their respective purpose?
3. The size of a p-value depends on the sample size $n$. How can this affect the interpretation of the p-value itself, and therefore of the analysis results?